

Curved-crease folding of bending-active plates as formwork

a reusable system for shaping corrugated concrete shell structures

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ABSTRACT

The critical environmental impact of the concrete construction industry demands material-efficient structures and construction methods applicable to low-tech and high-tech contexts. Thin structurally-informed shells with corrugations as stiffeners are material-efficient solutions as they gain their strength through their non-standard geometry. However, their bottleneck lies in their costly and wasteful formwork systems.

This research introduces curved-crease folding (CCF) of bending-active plates as a flexible, lightweight, and reusable formwork system for shaping corrugated concrete shell structures. CCF is extended to an initially closed configuration that unfolds initially-planar bending-active strips into a 3D formwork when actuated on-site. The curved creases control the shape and structurally stiffen the formwork shaping a concrete shell structure with stiffening corrugations.

The paper concentrates on the system design covering theoretical, computational, and fabrication aspects. The primary focus for the computational methods lies in implementing and extending the reflection method for the initially closed CCF; for the materialization method in textile hinge solutions for the curved creases. The approach is demonstrated with a small-scale proof-of-concept prototype.

The proposed system offers a material-efficient, self-supporting formwork solution that can be flat-packed for transport, rapidly erected on-site through the actuation of the CCF mechanism, and reused after concreting and decentering. The proposed formwork's geometry is not sensitive to stiffness variations as it is constrained by the CCF. Furthermore, the CCF makes the formwork independent of advanced machine technology, thus allowing for the construction of complex customized shapes also in low-tech contexts.

1 Unfolding of the curved-crease-folded bending-active formwork.

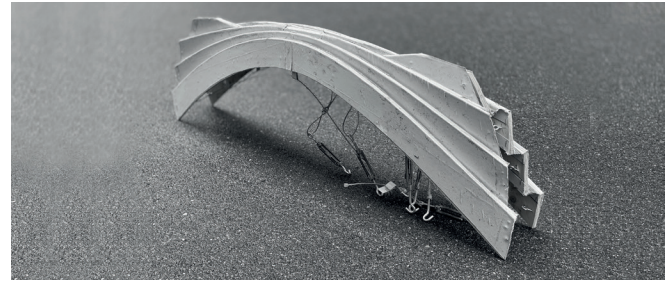
INTRODUCTION

Facing the critical impact of the AEC sector on the climate crisis, resource depletion, and waste production, we must decarbonize not only building structures but also their construction methods. Moreover, for a global impact, sustainable construction solutions must be broadly applicable and accessible not only in high-tech but also low-tech construction contexts. Innovation is particularly important for the widespread concrete construction industry with its large impact on global CO₂ emissions (Lehne and Preston 2018). Rethinking its conventional design and construction methods that rely on redundant material placement in standard slab and beam typologies offers great potential. Instead, thin funicular shell structures with corrugations as stiffeners offer material-efficient solutions as they gain strength through their structurally-informed geometry (Block et al. 2020). However, shaping such non-standard concrete structures poses a bottleneck. Conventional formwork systems for custom shapes are typically high in cost, material, and waste and limited to high-tech machining, such as CNC-milled timber or foam with GFRP coating (Kudless et al. 2020) or plywood plates mounted onto waffle substructures (Peri 2020).

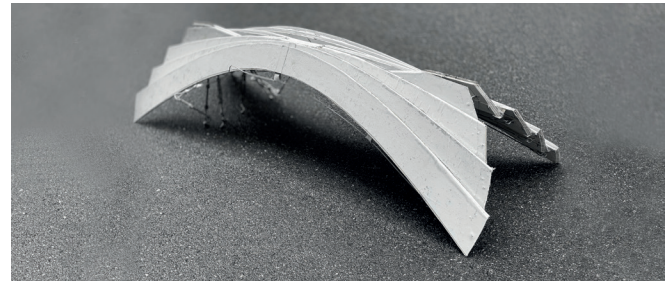
Alternative formwork solutions proposed in research range from additive manufacturing to flexible formworks (Jipa et al. 2019, Veenendaal et al. 2011). Flexible formworks base their efficiency on the structurally-informed geometry of their form-active structural system. Such systems are made from fabric shuttering together with a tensile cable-net falsework (Mendez et al. 2019, Popescu et al. 2020) or a bending-active falsework (Cuvilliers et al. 2017, Scheder-Bieschin et al. 2022). Shuttering and falsework are installed in consecutive steps and are not walkable.

The research presented aims to introduce a flexible formwork system of combined formwork and falsework. It proposes using curved-crease folding (CCF) of bending-active plates as a formwork system for shaping corrugated concrete shell structures. The combination of CCF and active bending is a bilateral mechanism where one actuates and amplifies the other. Planar strips are connected alongside curved-crease hinges and fold or unfold into a globally doubly-curved geometry serving as formwork (Figure 2^{a, b, c}). The curved creases control the shape and structurally stiffen the formwork, consecutively shaping the concrete shell structure with stiffening corrugations (Figure 2^d).

The proposed system would offer a material-efficient formwork solution that can be flat-packed for transport, fast erected on-site, immediately walkable, and reusable. The primary material is plywood, typical for conventional formworks. Since its sophistication lies in the geometric principle of the CCF rather than in advanced machine technology, it allows for custom shapes in low-tech contexts. The design development of such a construction system must be approached holistically,



2^a



2^b



2^c



2^d

integrating aesthetic, structural, and fabrication opportunities and constraints.

This paper focuses on the novel system design with its underlying concepts and proof-of-concept workflow from design to fabrication. The core of the system design is the CCF's shape control and stiffening for the formwork and resulting shell. Further, this research introduces an extended CCF formulation that starts from a closed configuration and unfolds into a 3D geometry. In the proof-of-concept pipeline, the paper primarily addresses the computational methods of the extended, initially closed CCF configuration in the COMPAS framework (Van Mele et al. 2017-2022), as well as the materialization challenge of the curved creases with a textile hinge strategy. The process is demonstrated through a small-scale prototype.

BACKGROUND

Curved-crease folding (CCF) is a popular research stream for its intriguing design space and sophisticated computational modeling methods. Huffman (1976), Duncan and Duncan (1982), and Fuchs and Tabachnikov (1999) established theorems on the behavior of CCF based on differential geometric analysis and introduced the method of reflection for modeling the CCF based on basic geometric principles. More recently, there have been many approaches to advanced computational simulation and optimization of CCF (Kilian et al. 2008, Bhooshan et al. 2015, Rabinovich et al. 2019). To simulate mechanical behavior, finite element analysis (FEA) simulation with the commercial software SOFiSTiK allows computing internal bending stresses for curved plates based on the unstressed state so that the system equilibrates with non-linear, third-order analysis away from the ideal target towards its deformed equilibrium shape (Bellmann 2017).

CCF finds application in non-structural architectural installations as kinetic façades or as foldable utility objects because of its simple fabrication of elegant shape from developable strips, its amplified actuation behavior, and its flat-packed transportability (Bhooshan et al. 2015, Choma 2021, Körner et al. 2016, Frommelt 2011). Bhooshan et al. (2015) utilized CCF as molds for shaping concrete for small prefabricated nodal segments assembled into a rib structure. These examples are materialized with thin sheets. Recent examples extend the use of CCF with thicker plates to structural applications (Maleczek et al. 2020, Basnak et al. 2020). At such scales, the plates are considered bending-active, and the curved creases are materialized with hinges between non-continuous plates.

Active bending is a method to elastically bend slender planar elements, such as splines or plates, into curved shapes without formworks (Lienhard 2014). It offers easy deployment, advantageous packaging, and lightweights. It allows constructing globally doubly-curved structures from planar plates with reduced waste, like in the historical example of Fuller (1959) and recent research (Schleicher and La Manga 2016). Bending-active structures suffer the dilemma of flexibility for forming and stiffness for structural performance to withstand external loads. Stiffening strategies are built-in tension cable elements (Takahashi 2016) or could be curved-crease folds resulting in corrugated sections that increase the moment of inertia as in conventional folded plate structures.

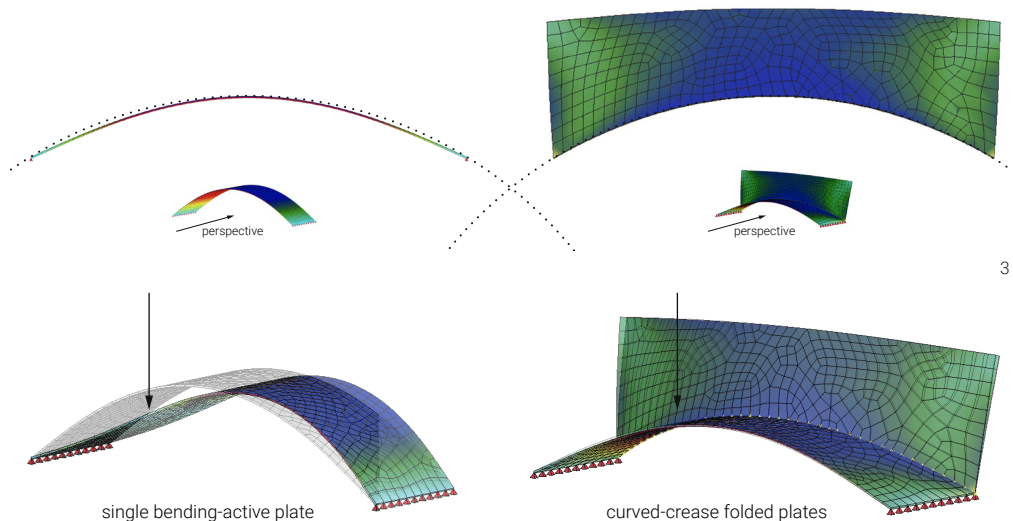
Curved folded plate structures have been constructed either from actively or passively bent separate plates whose edges are then interlocked in the curved state (Buri et al. 2011, Correa et al. 2016, Robeller et al. 2014) or by the assembly of the plates in their flat state and then actively bending and folding, actuated and shape-controlled by the curve-folding mechanism (Maleczek et al. 2020). The latter is fast, practical, and reversible. However, the foldability poses geometric limitations in the design space. This folding mechanism requires flexible hinges along the curved folds, such as textile hinges (Maleczek et al. 2020, Basnak et al. 2020). The separate plates enable starting from a closed configuration as identified in a single crease in the prototype of Basnak et al. (2020). For flat crease patterns, the folding of nonzero crease patterns to avoid self-collision is of particular interest (Ku and Demaine 2016).

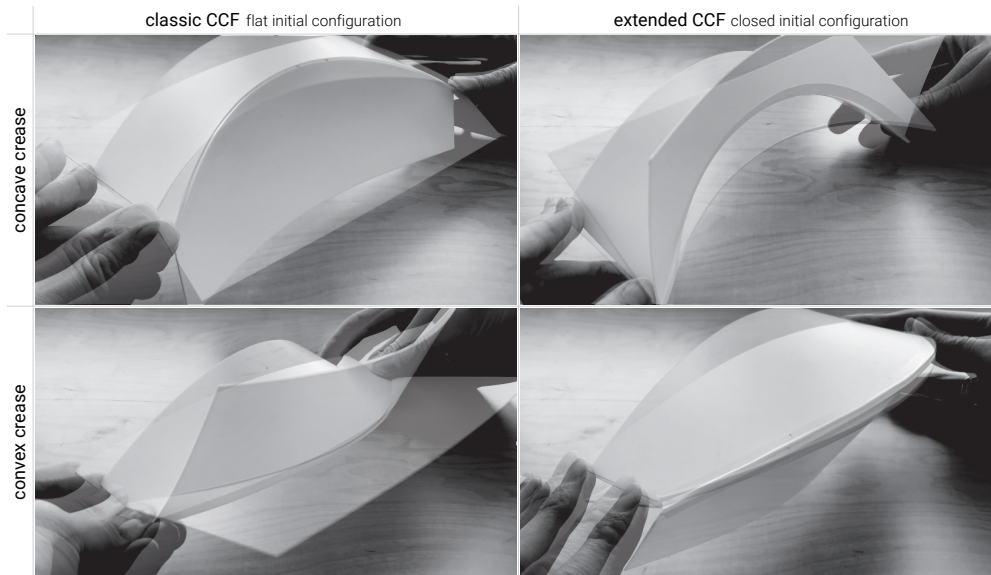
With the discussed potentials and limitations, the applicability of an extended CCF mechanism with bending-active plates and textile hinges as self-supporting formwork and falsework are investigated in the presented research.

2 Curved-crease-folded bending-active formwork in its closed ^a, half-unfolded ^b, and fully-unfolded state ^c, and the resulting corrugated concrete shell structure ^d shown on the proof-of-concept prototype.

3 Shape control comparison of a single bending-active plate and curved-crease folded plates with its crease curvature designed for constant bending curvature to a circular arc (dotted line) with FEA.

4 Stiffness comparison of a single bending-active plate to curved-crease folded plates by deformation under asymmetric wet-concrete load with FEA.





5 The four configurations of CCF in its classic and extended version with concave and convex creases showcased in paper models overlaying the starting and actuated positions.

5

SYSTEM DESIGN WITH UNDERLYING PRINCIPLES

The system design of the formwork and the resulting shell relies primarily on the shape control of CCF and the stiffening effect of CCF that translates to the concrete shell. Built-in restraining cables achieve further stiffening and actuate the system's unfolding. The system designs are based on two types of CCF, the classic version and an extended variation that offer flat-packed formworks that unfold like an accordion.

Shape control by curved creases of the formwork

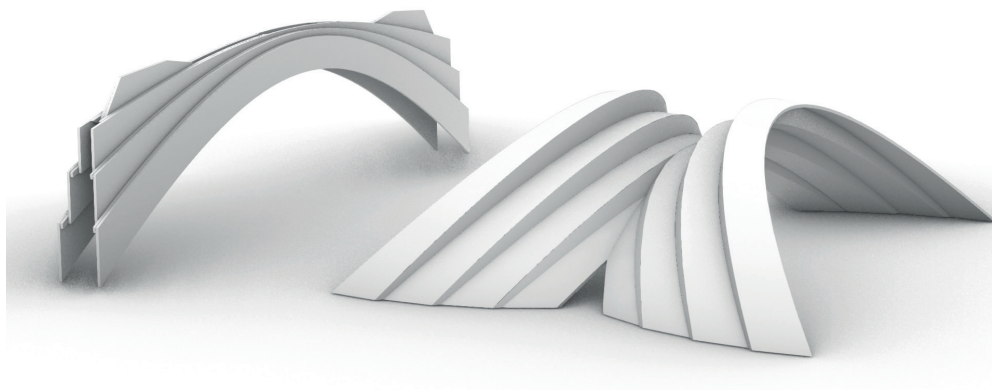
The curved creases control the equilibrium shape of the bending-active plates. Naturally, a single plate of constant stiffness actively bends into an Euler Elastica curve (Lienhard 2014). In CCF, the curvature of a curved crease controls the curvature of its adjacent plates; thus, a wide range of shapes can be achieved. In the example of Figure 3, the crease's curvature is computed with the reflection method such that the curvature of the plate satisfies a perfect arc. It is demonstrated by a form-finding simulation with the FEA-software SOFiSTiK, modeling

the mechanical bending resistance of the plates. The shape control only holds true in the proximity of the curved crease. Thus, the plates must be controlled by creases on both sides.

Stiffening by curved creases of the formwork and shell

A further advantage of the CCF is the stiffening effect on the formwork. The formwork must carry asymmetric loading of the wet-concrete weight. Compared to a single plate, the adjacent plate acts as a restraining diaphragm wall such that deformations decrease and stability increases dramatically, as demonstrated with the FE simulation (Figure 4).

The CCF shape results in a corrugated shell in concrete and the stiffening scheme is translated to the shell structure. The corrugations increase the static height, i.e., the moment of inertia of the section, and if the concrete shell is form-found around an envelope of thrust lines for asymmetric loads, it can result in a funicular, compression-only concrete structure, not in need of structural reinforcement (Block et al. 2020).



6 Closed flat-packed formwork (left) and the globally doubly-curved CCF formwork (right) after unfolding like an accordion, shown on prototype design geometry.

6

Concept of classic CCF and the initially closed CCF

Curved-crease folding starts from a flat, continuous sheet or thicker plates cut and re-joined with a hinge. During actuation, as the adjacent plates fold towards each other, they bend with single curvature in opposing directions. As an extension, this research introduces an extended CCF that starts from a flat-packed, initially closed configuration. The plates are non-continuous and mirrored along their creases. Contrary to the classic CCF, the plates unfold away from each other and the system opens up. The plates' curvature is oriented away from each other for a concave crease and towards each other for a convex crease. Figure 5 shows the four basic types of the classic and extended CCF with concave and convex creases, respectively.

The classic CCF offers the advantage of being flat and the downside of a large format and is thus considered more suitable for prefabrication. In contrast, the extended CCF offers the advantage that its flat-packed pile is practical for transportation and opens up like an accordion into 3D geometry (Figure 6), hence considered more suitable for in-situ applications.

Further, introducing the extended CCF broadens the constrained design space of the classic CCF. However, it demands modified computational modeling methods and poses the challenge of the hinge materialization in the closed, obstructed state.

COMPUTATIONAL MODELING METHODS

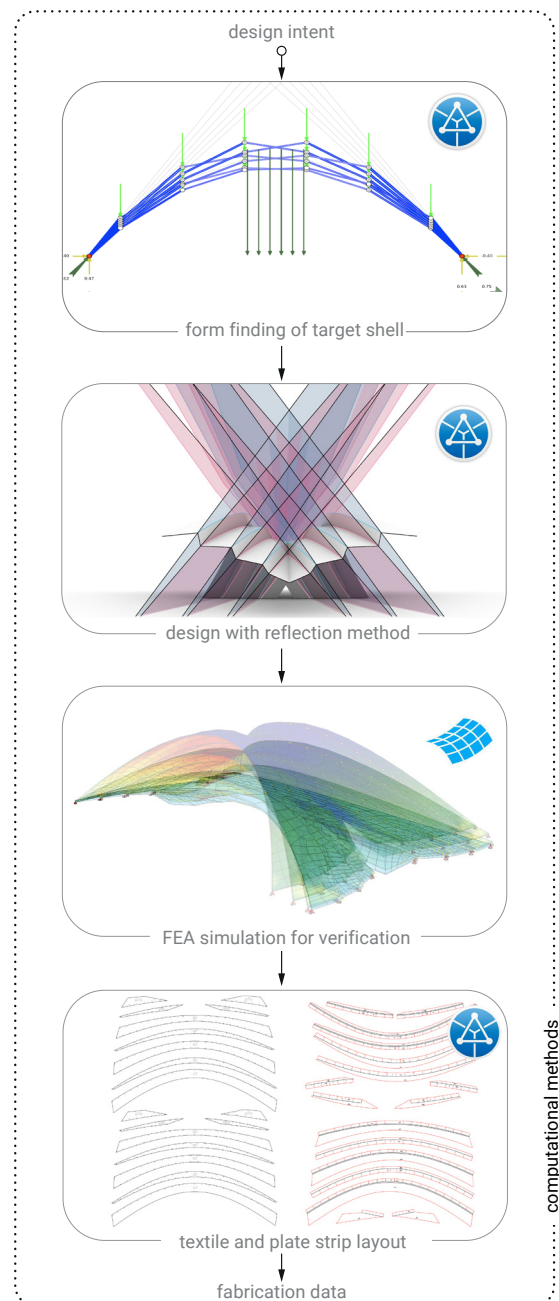
Computational workflow

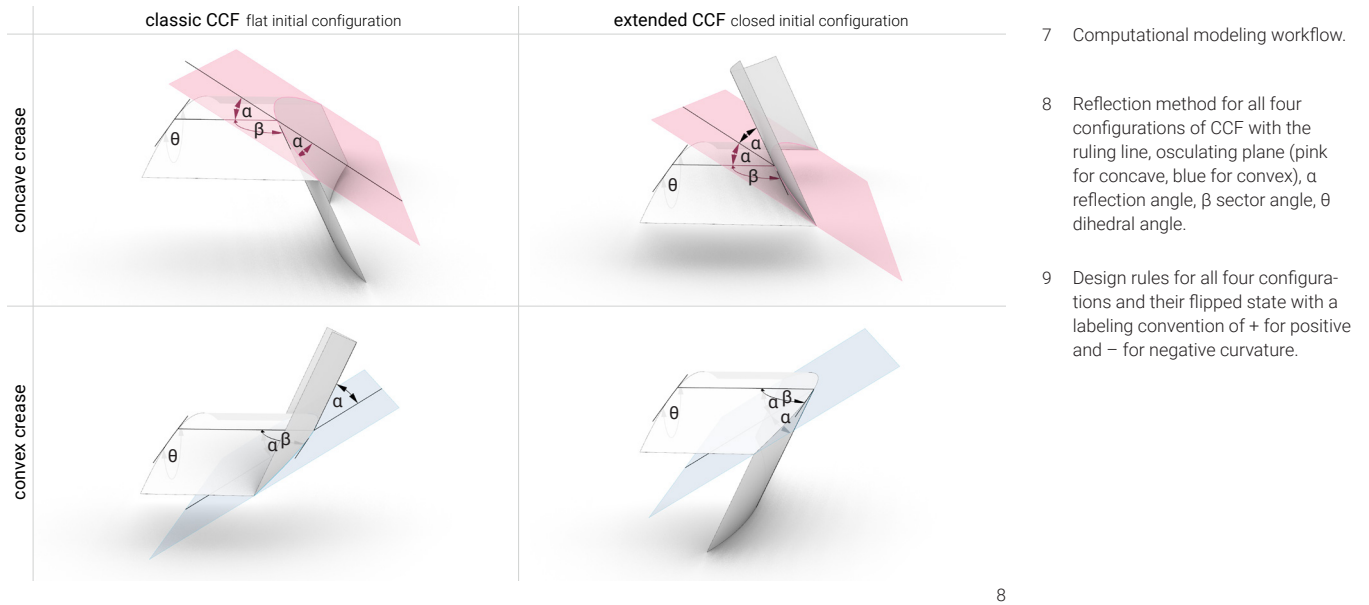
The proposed construction system requires a computational modeling framework that simultaneously integrates the geometric, structural, and fabrication constraints for both the formwork and the resulting shell. The computational methods are implemented in COMPAS, an open-source framework for research in AEC (Van Mele et al. 2017-2022). The design workflow is shown based on a design that serves for the proof-of-concept prototype. The workflow (Figure 7) commences with the funicular form finding of the concrete shell that defines the target geometry for the formwork. The design exploration toward this target geometry for the CCF formwork is implemented with the geometric reflection method. The CCF geometry is verified and analyzed with a form-finding simulation through a COMPAS interface with SOFiSTiK. Finally, the cutting pattern for the textile and plate strips is generated and exported as fabrication data from COMPAS. This paper focuses on the reflection method for the closed CCF; the other workflow steps will be discussed in a complementary paper.

Method of reflection

The theorems for the reflection method and the differential geometry for the classic CCF can be applied with modification to the closed CCF. They define the interrelationships among

angles associated with creases and curvature definitions of the developable surfaces. Figure 8 shows all four configurations for cylindrical surfaces with parallel ruling lines. The ruling lines are where all points on the surface share the same tangent plane and indicate the direction of zero Gaussian curvature. These are reflected along osculating planes with the reflection angle α . The sector angle β that defines the crease curvature correlates with the dihedral angle θ that defines the plate bending curvature. In the classic flat case, the continuous plate, which is a reflection of the initial strip, is reflected along the osculating plane. In contrast, in the extended closed case, the identical plate is





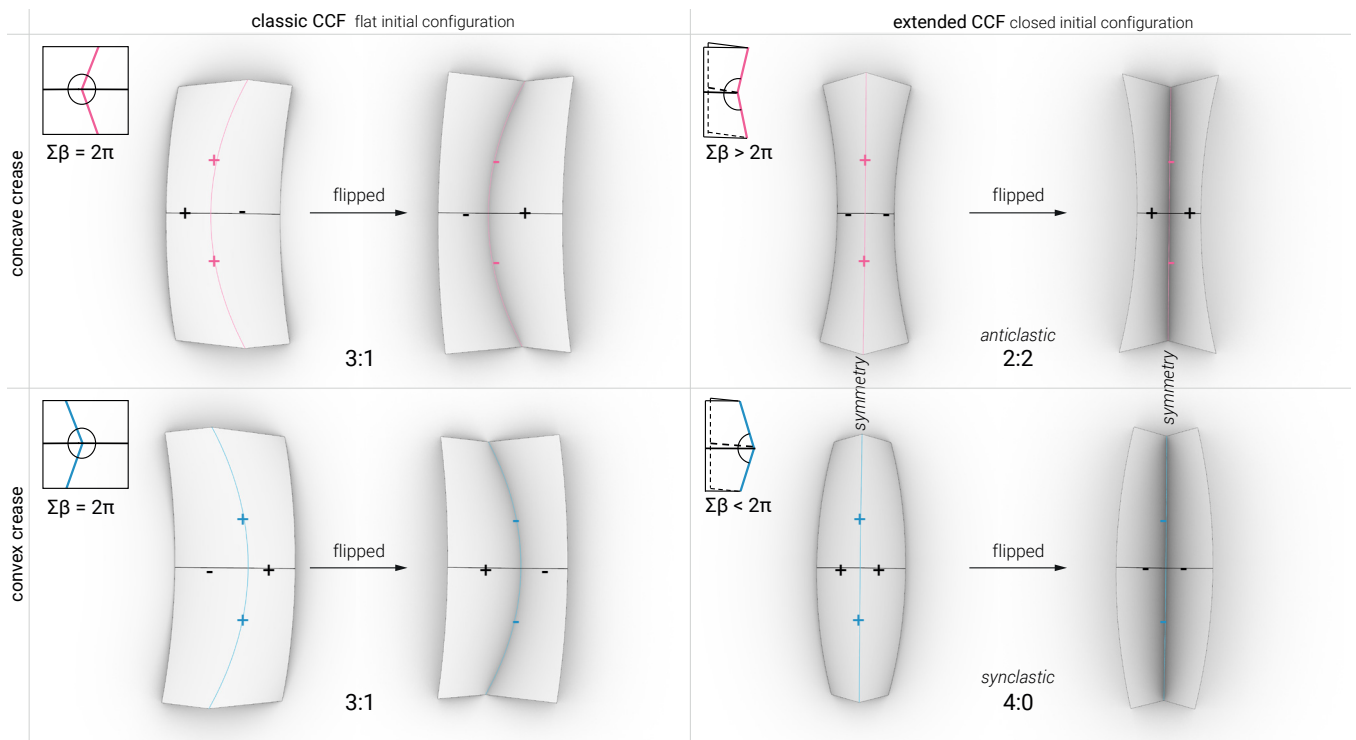
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reflected along the osculating plane as they are directly mirrored. The reflection method is limited to planar creases, resulting in a constant reflection angle over the entire crease; however, the angle normal to the crease varies unless for special cases.

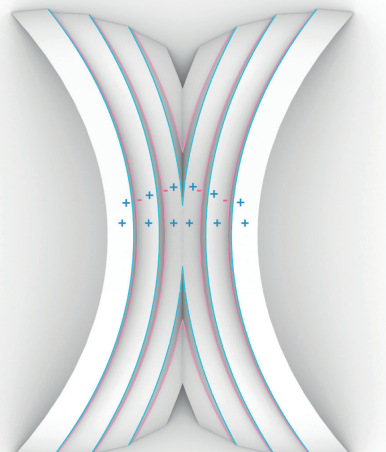
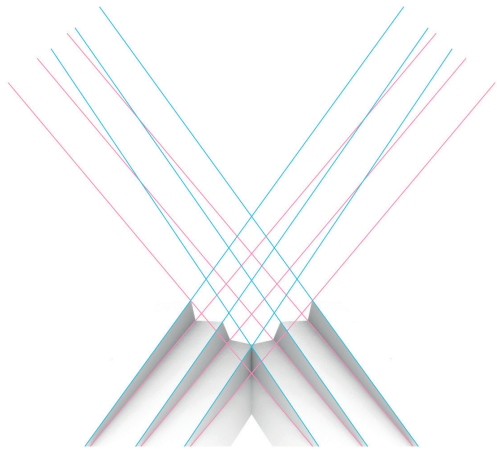
The reflection method is implemented for both the classic and the extended CCF in the COMPAS framework with planar quad meshes. Further, the implementation includes the simulation of the folding and unfolding process.

Design rules with the reflection method

In the classic CCF of developable continuous plates, the sum of all plane sector angles $\Sigma\beta$ remains constant at exactly 2π at all times during folding (Figure 9 small diagram). However, in the extended CCF, the plates are cut out and re-joined with a sum of all plane sector angles $\Sigma\beta$ greater than 2π for the concave crease and smaller than 2π for the convex crease. These sector angles also remain constant at all times during unfolding.



9



10 Application of the rules to the prototype design with osculating planes of convex (blue / +) and concave (pink / -) creases and curvature of plates (italic +).

10

Huffman (1976) introduced a labeling convention with + for positive and - for negative curvature. This is useful for analyzing the resulting shape and the possibilities of combining multiple strips. In the classic CCF, the crease and ruling lines must always be in a ratio of 3 + to 1 - and vice versa. This results in shapes around the crease that are neither anti- nor synclastic, as it still is formed from a continuous developable surface. This research extends the convention for the closed CCF - for the concave crease, the ratio is 2 + and 2 - alternating, and for the convex crease, the ratio is either 4 + or 4 -. This leads to anti-clastic and synclastic curvatures, respectively, aligning with the sum of sector angles (Figure 9 *render diagrams*). In the plan view, the closed CCF is symmetric along its crease due to the reflection of the identical plate, whereas the classic CCF is antisymmetric due to the reflection of the reflected side.

These rules are applied to the computational design of the physical prototype (Figure 10). The design consists purely of extended CCF creases and alternates in ridges along concave creases and valleys along convex creases. Its global shape is defined by variable osculating plane inclination and distance, where each orientation affects the subsequent creases. The osculating planes are rotated around parallel axes and translated such that the boundary and rise curve inwards, resulting in a globally doubly-curved geometry. In elevation, this results in variable corrugation widths wider towards the supports and almost acute towards the ends.

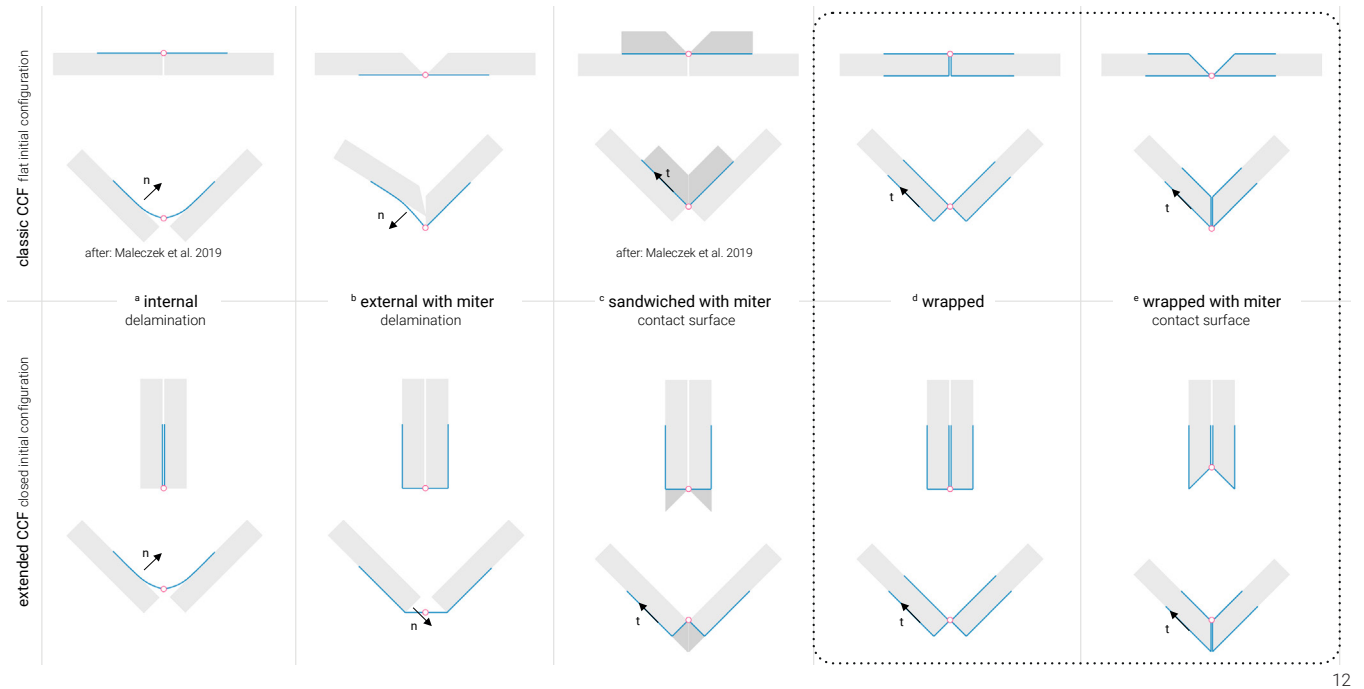


11 Materialized open and closed CCF with classic ^a and extended ^b initial configuration and cable-actuated folded ^c and unfolded ^d 3D-geometry.

12 Textile hinge strategies with one-sided, sandwiched, and two-sided/wrapped laminated fabric (blue) with sewing seam at the textile hinge position (pink), with normal (n) or tangential (t) acting force in glue interface (shown for one side only, but symmetrical).

13 Integrated cable attachment loop into textile hinge during unfolding.

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MATERIALIZATION METHODS

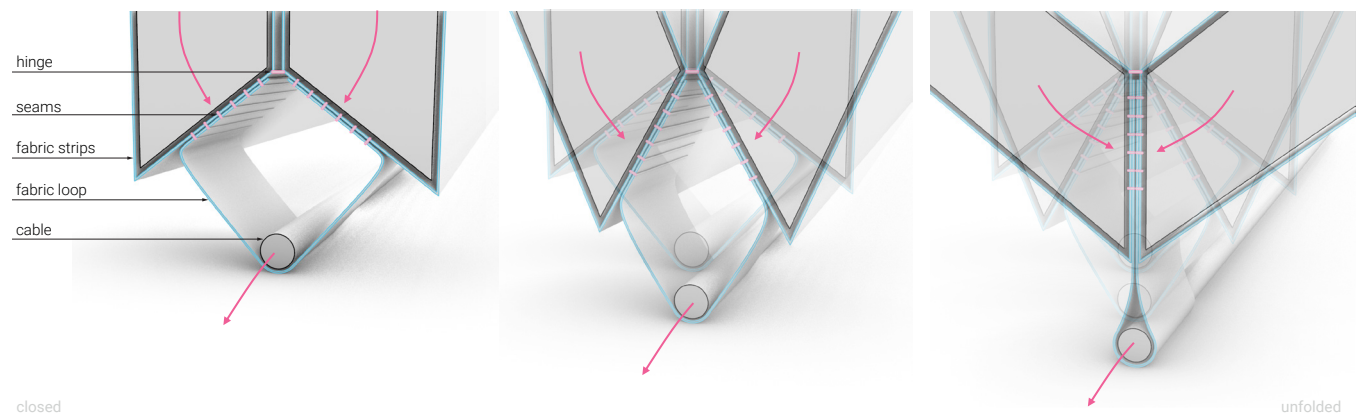
The aim of this research is to develop a practical and reusable formwork system applicable to a wide range of contexts. The main challenge is the materialization of the curved hinges, using broadly available materials, with a fabrication workflow that is feasible in low-tech as well as high-tech contexts, adjustable to the availability of labor or technology. This work predominantly focuses on a low-tech approach to oppose the common state-of-the-art high-tech manufacturing strategies for custom shapes. It is demonstrated on the proof-of-concept prototype that materializes the computationally designed proposal.

Textile hinges

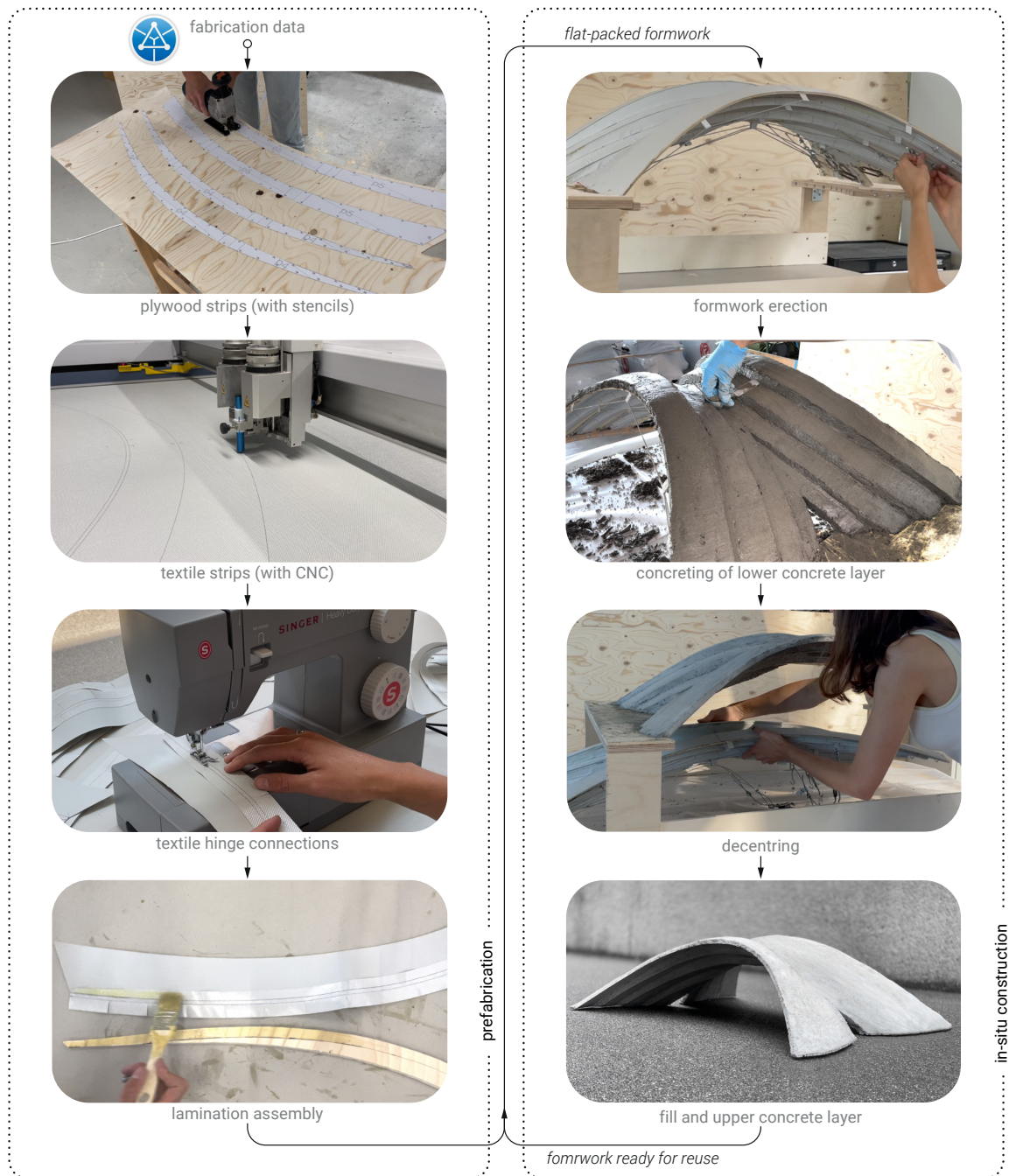
The curved creases are materialized with textile hinge strips laminated to plates. The hinge strategy is developed to prevent delamination, obtain a hinge position compatible with a nonzero thickness model, and achieve a joining strategy for the closed

CCF configuration. As the inner side of the plates in the closed CCF is not accessible during the prefabrication, a joining strategy must be chosen that does not require tool access as for mechanical fasteners. Instead, lamination with a bonding agent allows connecting a textile to the plates by applying external pressure without the need for internal access between the plates and textile.

A textile hinge laminated to a plate only on one side is at risk of peeling off when subject to forces normal to the plate (Figure 12^{a,b}). To prevent this failure, Maleczek et al. (2019) demonstrated that this can be avoided by clamping the textile with an additional slat (Figure 12^c). The limitation of this solution is that it requires miter joints and that the slats increase the sectional height decreasing the minimal allowable curvature. Building upon this strategy, this research proposes laminating textile to both sides of the plate connected at the hinge position with a



13



14

sewn seam, sandwiching the plate in the middle. As a result, the forces normal to the plate are redirected to the opposite side and taken by tangential forces in which a laminated connection performs best (Figure 12^{d,e}).

In the closed CCF, the hinge pole must be positioned at the contact surface to the adjacent plate so that the plates can freely rotate around it when the CCF system unfolds. When materialized with a miter cut, the rotation is locked to the angle where contact forces block the CCF unfolding. This offers additional stiffness to the curve folded formwork. Figure 11

shows the applied strategy of the wrapped hinge with miter joint (Figure 12^e) for both the classic and extended CCF.

The custom-sewn textile hinges allow integrating features such as accommodating attachment loops for the actuation cables (Figure 2). These loops can be sewn into the seam simultaneously with the connection of two adjacent textile strips. If the loop is connected at the inside of the miter joint, the longitudinal tensioning of the cable to unfold the formwork supports the unfolding as it pulls together the abutting faces without inducing delaminating forces normal to the surface (Figure 13).

Formwork material choice

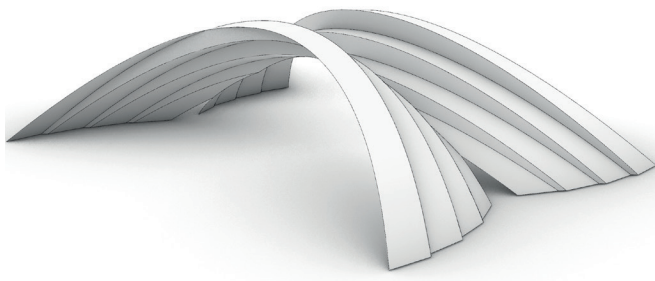
In the physical prototype, the bending-active strips are spruce plywood plates of 5mm thickness. Plywood is commonly used for conventional formworks. Furthermore, it is particularly suitable for active bending with its lightweight property of approximately 2.5 kg per m² and its ratio of flexural strength to stiffness greater than 2.5×10^{-3} . The orthotropic material is oriented such that the stiffer direction is along the ruling direction and the weaker direction along the bending direction. The orthotropic layout is particularly compatible with cylindrical surfaces where the rulings are parallel, which is not the case for conical or tangent developable surfaces.

For this prototype, the textile hinge strips are materialized as one-sided PVC-coated woven polyester textiles. The strips are designed as wide as to cover the plywood plates entirely to simultaneously serve as a protection layer for the plywood from the concrete and offer a smooth demoldable surface on the PVC-coated side. The non-coated side is crucial for adherence to the plates.

Fabrication workflow

The fabrication workflow comprises the prefabrication of the formwork and the in-situ construction (Figure 14). The workflow is demonstrated on the small-scale physical prototype, testing the system design and investigating the logistical steps of the process.

In the prefabrication, first, the plywood and textile pieces are cut and labeled based on the fabrication data from the design process. Depending on the context and availability of technology, these prefabrication tasks can either be executed by labor with paper stencils or using CNC machines. Second, the textile strips of both sides of a hinge are connected with a sewing machine, integrating the cable attachment loops. Third, the plate and textile strips are assembled with adhesive connections using a high-strength bonding agent. Lastly, the cables are attached by threading through the loops along the hinge lines and anchored to the plates at their ends. The resulting prefabricated element is compact, flat-packed, and relatively lightweight, practical to be transported to the site also on larger scales.



15^a

The erection of the in-situ formwork is actuated by turnbuckles in the contracting cables that pull the supports towards each other and actuate the bending of the plates together with the unfolding of the creases. Through this bilateral mechanism, the formwork system deforms from a compact folded-up into an articulated 3D globally double-curved geometry (Figure 1). Onto this articulated formwork, the concrete is directly applied. The concrete is a regular mix with a low water-cement ratio of 0.37 for high viscosity and small aggregates for workability. The developable geometry of the strips offers the advantage that the concrete can be shaped following the ruling lines with a straight tool. This step creates the lower layer of the double-layered sandwich concrete shell. Before or after decentering, a low-strength, low embodied-carbon fill and upper concrete layer are applied, resulting in a double-layered sandwich shell. The decentering of the removable formwork is carried out by first lowering the formwork slightly to avoid collision with the concreted shell and then refolding it. The formwork could be immediately reused for another concrete shell.

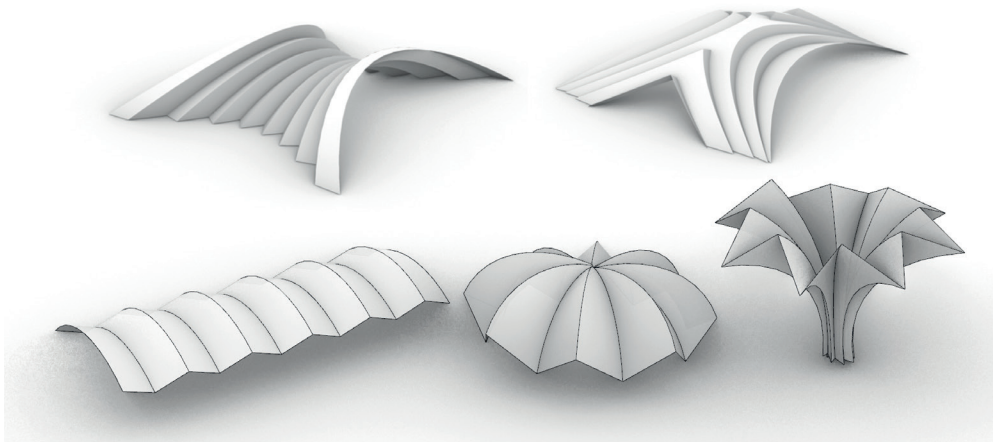
RESULTS AND DISCUSSION

The proposed computational and materialization methods were demonstrated with a proof-of-concept digital model and physical prototype, revealing the potentials and limitations of bending-active CCF for formwork systems for efficiently constructing corrugated concrete shells. The resulting physical prototype with a span of 1 meter is congruent in shape to the form-found digital model (Figure 15). This demonstrates that the computational design process and the fabrication methods suit the proposed formwork system. The system design allowed for the exploration of a custom geometry in the design process as well as its realization without the need for advanced technology.

The materials used in the prototype are low-cost and broadly available. The quality of the plate must not be high, as the geometry is controlled by the robust CFF system. For example, a plate strip of the physical prototype was assembled from two plates joined in the middle. Its shape displayed a kink along its connection line when bending the plate independently. However, when later integrated into the CCF prototype, no kink could be observed as the strip was well constrained by the curved



15^b



- 14 Fabrication workflow - from prefabrication to construction.
- 15 Computational geometry ^a in comparison to the physical prototype ^b of the first concrete shell layer.
- 16 Design possibilities for alternating convex and concave initially-closed creases (extended CCF) with parallel translation or rotation of reflection planes.

16

creases to its neighboring plates that impose the curvature. However, in future work, other material choices, such as GFRP for CCF like Körner et al. (2016) or Choma (2021), could be investigated to offer a robust and mono-material formwork system for reusability in high-tech contexts.

The formwork was load tested with a tenfold (25 kg) of its self-weight (2.5kg) and supported the wet-concrete weight with minor deflections. This demonstrates the stiffening effect of the folded plates and the restraining cables. In contrast to textile formwork systems, this stiff formwork could be immediately walkable also at larger scales. The formwork system is the self-supporting, load-bearing falsework and the concrete-shaping shuttering in one. This could be a further advantage over the textile formwork systems consisting of separate falsework and shuttering as it could dramatically simplify the on-site logistics.

The purpose of the small-scale physical prototype was for proof of concept. Future research on system development, structural design, and prototypes will focus on upscaling the system to make it applicable to the construction industry. Critical limitations lie in the implications on the fabrication and the structural integrity, including admissible forces for the textile hinges and stability of the plates while unfolding and supporting the wet-concrete weight. Extra stiffness in the non-bending direction could be achieved by extra stiffeners, as Basnak et al. (2020), or by subtractive processing of thicker plates along the rulings. In addition to the actuation cables, gravity could drive the actuation. Further, fabrication at larger scales must consider the limited available sizes of plates by either joining multiple plates as the stiffness variation does not impact the CCF-controlled shape or by dividing the spanning direction with curved creases resulting in more intricate layouts.

Such intricate layouts demand further development of computational methods. These could also enable complex geometries with parallel translation and rotation of the osculating planes, inclinations of the planes around varying axes, combining the classic and the extended CCF, including cone and tangent surfaces, and, lastly, non-planar creases. This would enlarge the design space of the system dramatically. However, the potential collision of the formwork with the concrete shell during decentering limits the range of geometries. Already introducing the extended CCF with planar creases enabled a wide range of custom shape opportunities (Figure 16).

CONCLUSION

The proposed formwork system for shaping corrugated concrete shells allows for a high degree of prefabrication, flat-packed transport, and rapid on-site deployment through the actuation of the CCF mechanism and the integration of falsework and shuttering. It is walkable, reusable, and self-supporting. The system requires neither high material consumption nor high technology as its structural strength and fabrication strategy rely on its geometrical constraints. Its independence from advanced machine technology allows for the construction of complex customized shapes also in low-tech contexts. If scaled up, the proposed construction system could find applications in corrugated shell structures for vaulted floors, roofs, bridges, and funnel-shaped columns. This would bring CCF out of the research realm toward an application that addresses the construction industry's critical challenges in developing and industrialized construction contexts.

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IMAGE CREDITS

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